Paper Reference(s)

6684/01

Edexcel GCE

Statistics S2

Advanced Level

Monday 22 June 2015 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Statistics S2), the paper reference (6684), your surname, other name and signature.
Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has 7 questions.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P44846A

1. In a survey it is found that barn owls occur randomly at a rate of 9 per 1000 km^2.

(*a*) Find the probability that in a randomly selected area of 1000 km^2 there are at least 10 barn owls.

(2)

(*b*) Find the probability that in a randomly selected area of 200 km^2 there are exactly 2 barn owls.

(3)

(*c*) Using a suitable approximation, find the probability that in a randomly selected area of 50 000 km^2 there are at least 470 barn owls.

(6)

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2. The proportion of houses in Radville which are unable to receive digital radio is 25%. In a survey of a random sample of 30 houses taken from Radville, the number, X, of houses which are unable to receive digital radio is recorded.

(*a*) Find $\mathrm{P}(5 \leq X < 11)$.

(3)

A radio company claims that a new transmitter set up in Radville will reduce the proportion of houses which are unable to receive digital radio. After the new transmitter has been set up, a random sample of 15 houses is taken, of which 1 house is unable to receive digital radio.

(*b*) Test, at the 10% level of significance, the radio company's claim. State your hypotheses clearly.

(5)

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3. A random variable X has probability density function given by

$$\mathrm{f}(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ k\left(1 - \dfrac{x}{6}\right) & 2 < x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(*a*) Show that $k = \dfrac{1}{4}$.

(4)

(*b*) Write down the mode of X.

(1)

(*c*) Specify fully the cumulative distribution function $\mathrm{F}(x)$.

(5)

(*d*) Find the upper quartile of X.

(4)

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4. The continuous random variable L represents the error, in metres, made when a machine cuts poles to a target length. The distribution of L is a continuous uniform distribution over the interval [0, 0.5].

(*a*) Find $\mathrm{P}(L < 0.4)$.

(1)

(*b*) Write down $\mathrm{E}(L)$.

(1)

(*c*) Calculate $\mathrm{Var}(L)$.

(2)

A random sample of 30 poles cut by this machine is taken.

(*d*) Find the probability that fewer than 4 poles have an error of more than 0.4 metres from the target length.

(3)

When a new machine cuts poles to a target length, the error, X metres, is modelled by the cumulative distribution function $\mathrm{F}(x)$ where

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0 \\ 4x - 4x^2 & 0 \leq x \leq 0.5 \\ 1 & \text{otherwise} \end{cases}$$

(*e*) Using this model, find $\mathrm{P}(X > 0.4)$.

(2)

A random sample of 100 poles cut by this new machine is taken.

(*f*) Using a suitable approximation, find the probability that at least 8 of these poles have an error of more than 0.4 metres.

(3)

5. *Liftsforall* claims that the lift they maintain in a block of flats breaks down at random at a mean rate of 4 times per month. To test this, the number of times the lift breaks down in a month is recorded.

(*a*) Using a 5% level of significance, find the critical region for a two-tailed test of the null hypothesis that 'the mean rate at which the lift breaks down is 4 times per month'. The probability of rejection in each of the tails should be as close to 2.5% as possible.

(3)

Over a randomly selected 1 month period the lift broke down 3 times.

(*b*) Test, at the 5% level of significance, whether *Liftsforall*'s claim is correct. State your hypotheses clearly.

(2)

(*c*) State the actual significance level of this test.

(1)

The residents in the block of flats have a maintenance contract with *Liftsforall*. The residents pay *Liftsforall* £500 for every quarter (3 months) in which there are at most 3 breakdowns. If there are 4 or more breakdowns in a quarter then the residents do not pay for that quarter.

Liftsforall installs a new lift in the block of flats.

Given that the new lift breaks down at a mean rate of 2 times per month,

(*d*) find the probability that the residents do not pay more than £500 to *Liftsforall* in the next year.

(6)

6. A continuous random variable X has probability density function f(x) where

$$f(x) = \begin{cases} kx^n & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where k and n are positive integers.

(a) Find k in terms of n.

(3)

(b) Find E(X) in terms of n.

(3)

(c) Find E(X^2) in terms of n.

(2)

Given that $n = 2$,

(d) find Var(3X).

(3)

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7. A bag contains a large number of 10p, 20p and 50p coins in the ratio 1 : 2 : 2.

A random sample of 3 coins is taken from the bag.

Find the sampling distribution of the median of these samples.

(7)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme		Marks
1. (a)		**notes**	
	$\text{P}(N \geq 10) = 1 - \text{P}(N \leq 9)$	M1: using or writing $1 - \text{P}(N \leq 9)$ or $1 - \text{P}(N < 10)$	M1 A1
	= 0.4126	A1: awrt 0.413	
(b)	Y represents number of owls per 200 km^2 $\Rightarrow$ $Y \sim \text{Po}(1.8)$	B1: using or writing Po(1.8)	B1
	$\text{P}(Y = 2) = \dfrac{e^{-1.8}1.8^2}{2!}$	M1 : for a single term of the form $\dfrac{e^{-\lambda}\lambda^2}{2!}$ with any value for λ or $\text{P}(X \leq 2) - P(X \leq 1)$	M1 A1
	= 0.2678	A1: awrt 0.268	
(c)	Normal approximation	M1: Using or writing, normal approximation with mean = 450	M1
	$\mu = 50 \times 9 = 450 \quad \sigma^2 = 450$	M1: Using or writing the mean = variance. Does not need to be 450. May be seen in the standardisation calculation.	M1
	$\text{P}(X \geq 470) \approx 1 - \text{P}\left(Z < \dfrac{469.5 - 450}{\sqrt{450}}\right)$	M1: $\pm\left(\dfrac{(470 \text{ or } 469.5 \text{ or } 470.5) - \textit{their mean}}{\textit{their sd}}\right)$ May be implied by a correct answer or z = awrt 0.92	M1
		M1: dep on previous method mark being awarded. Using a continuity correction 470 ± 0.5 May be implied by a correct answer or z = awrt 0.92	dM1 A1
		A1: correct standardisation no need to subtract from 1. Award for $\dfrac{469.5 - 450}{\sqrt{450}}$ or awrt 0.92 or a correct answer	
	= 0.1788	A1: awrt 0.179	A1 **(6)**

Question Number	Scheme		Marks
2(a)		**notes**	
	$X \sim B(30, 0.25)$	B1: using B(30, 0.25)	B1
	$P(X \le 10) - P(X \le 4) = 0.8943 - 0.0979$	M1: using $P(X \le 10) - P(X \le 4)$ or $P(X \ge 5) - P(X \ge 11)$ oe	M1 A1
	$= 0.7964$	A1: awrt 0.796	
	NB a correct answer gains full marks		

(b)	$H_0 : p = 0.25$ $H_1 : p < 0.25$	B1: Both hypotheses correct, labelled H_0 or NH or H_n and H_1or AH or H_a , must use p or $p(x)$ or π	B1
	B(15, 0.25)	M1: for using B(15, 0.25)	M1 A1
	$P(X \le 1) = 0.0802$	A1: awrt 0.0802 or CR $X \le 1$ (allow $P(X \ge 2) = 0.9198$)	
	NB: Allow M1 A1 for a correct CR with no incorrect working		
	Reject H_0 or Significant or 1 1ies in the critical region	M1: A correct statement – do not allow contradictory non contextual statements. Follow through their Probability/CR (for 1 or 2 tail test). If no H_1 given then M0. Ignore their comparison. For a probabillity < 0.5, statement must be correct compared to 0.1 for 1 tail test and 0.05 for 2 tailed test or if the probability > 0.5, statement must be correct compared to 0.9 for 1 tail test and 0.95 for 2 tailed test.	dM1 A1cso
	There is evidence that the radio **company's** claim is true. **Or** The new transmitter will reduce the proportion of houses unable to receive **radio**	A1: cso (all previous marks awarded) and a correct statement containing the word **company** if writing about the claim or **radio** if full context.	

Question Number	Scheme	Notes	Marks
3(a)	$\int_0^2 kx^2\,dx + \int_2^6 k\left(1-\frac{x}{6}\right)dx = 1$	M1: for adding the two integrals, and attempting to integrate, at least one integral $x^n \to x^{n+1}$, ignore limits and does not need to be put equal to 1. Do **not** award if they add before integrating	M1 A1
	$k\left[\frac{x^3}{3}\right]_0^2 + k\left[x-\frac{x^2}{12}\right]_2^6 = 1$	A1: correct integration, ignore limits and does not need to be put equal to 1	
	$k\left[\frac{8}{3}\right] + k\left[3-\frac{5}{3}\right] = 1$	M1: dependent on first M being awarded, correct use of limits and putting equal to 1. This may be seen as F(2) = $\frac{8}{3}k$ and using F(6) = 1	dM1 A1cso
	$4k = 1$	A1: cso answer given so need $4k = 1$ leading to $k = \frac{1}{4}$	
	$k = \frac{1}{4}$ *		
NB Validation – if they substitute in k = ¼ you may award the 1st three marks as per scheme. For the Final A mark they must say " therefore k = ¼"			

(b)	2	B1: cao	B1

(c)	$\int_0^x kt^2\,dt = \frac{kx^3}{3}$	M1: attempting to find $\int_0^x kt^2\,dt$ $t^2 \to t^3$, ignore limits, may leave in terms of k	M1
	$\int k\left(1-\frac{t}{6}\right)dt = k\left[t-\frac{t^2}{12}\right]+C$ $= kt - k\frac{t^2}{12}+C$ F(6) =1 $6k-3k+C=1 \ \therefore C = \frac{1}{4}$	M1: attempting to find $\int k(1-\frac{t}{6})dt$ at least one integral $t^n \to t^{n+1}$ **and** either have + C $(C \neq 0)$ and use F(6) =1 **or** have limits 2 and x and + "their $\int_0^2 kt^2\,dt$" and attempt to integrate $t^n \to t^{n+1}$ NB: may use any letter, need not be t ,condone use of x	M1
	$F(x)\begin{cases} 0 & x<0 \\ \frac{x^3}{12} & 0 \le x \le 2 \\ \frac{x}{4}-\frac{x^2}{48}+\frac{1}{4} & 2 < x \le 6 \\ 1 & x > 6 \end{cases}$	A1: second line correct A1: third line correct B1: first and fourth line correct they may use "otherwise" instead of x < 0 or x > 6 but not instead of both	A1 A1 B1
	NB: Condone use of < rather than ≤ and vice versa		

Question Number	Scheme		Marks
(d)	$\frac{x}{4}-\frac{x^2}{48}+\frac{1}{4}=0.75$	M1: putting their line 2 or their line 3 = 0.75	M1 A1
	$x^2-12x+24=0$ oe	A1: The correct quadratic equation – like terms must be collected together	
	$x=\frac{12\pm\sqrt{144-4\times 24}}{2}$	M1d: dep on previous M1 being awarded. A correct method for solving a 3 term quadratic equation = 0 leading to x = …Use either the quadratic formula or completing the square - If they quote a correct formula and attempt to use it, award the method mark if there are small errors. Where the formula is not quoted, the method mark can be implied from correct working with values but is lost if there is a mistake. If they attempt to factorise award M1 if they have $\left(x^2+bx+c\right)=(x+p)(x+q)$, where $\lvert pq\rvert=\lvert c\rvert$ leading to $x=\ldots$ May be implied by a correct value for x	dM1 A1
	= 2.54 or $6-2\sqrt{3}$	A1: awrt 2.54 or $6-2\sqrt{3}$ or $6-\sqrt{12}$. If 2 values for x are given they must eliminate the incorrect one.	

Question Number	Scheme		Notes	Marks
			Notes	
4(a)	0.8		B1: cao	B1
(b)	0.25		B1: cao	B1
(c)	$\frac{(0.5-0)^2}{12} = \frac{1}{48}$ or awrt 0.0208		M1: for $\frac{(0.5\pm 0)^2}{12}$ or for $\int_0^{0.5} 2x^2\mathrm{d}x - (\text{their } (b))^2$ with some integration $x^n \to x^{n+1}$	M1A1
			A1: $\frac{1}{48}$ or awrt 0.0208 or awrt 2.08×10^{-2}	
(d)	P(L > 0.4) = 0.2	P(L < 0.4) = 0.8	An awrt 0.123 award B1 M1 A1	B1 dM1A1
	Y~ B(30, 0.2)	Y~ B(30, 0.8)	B1: using or writing B(30, their P(L < 0.4) or B(30, their P(L > 0.4) . If they have not written these probabilities in this part use answer from part (a) ie P(L < 0.4) = (a) or P(L > 0.4) = 1- (a)	
	P($Y\leq$ 3) = 0.1227	P($Y\geq$ 4) = 0.1227	M1: dependent on previous B mark being awarded. Using B(30,P(L>0.4) with P($Y\leq$ 3) written or used **Or** B(30 P(L<0.4)) with P($Y\geq$ 4) written or used A1: awrt 0.123	
(e)	$1-\left[4\times 0.4-4\times 0.4^2\right]=\frac{1}{25}$ or 0.04		M1: Using 1- F(0.4) or F(0.5) – F(0.4) or $\mathrm{P}(X \leq 0.5)-\mathrm{P}\left(X \leq 0.4\right)$. Must see some substitution of 0.4 A1: $\frac{1}{25}$ or 0.04 only	M1A1
(f)	Po(4)		B1ft: using or writing Po(4) **NB** for ft they must either write 100 × “their 0.04” and use Poison or write Po(“their λ ”) Allow P instead of Po	B1ft
	P($X\geq$ 8) = 1- P($X\leq$7)		M1 using or writing 1- P($X\leq$7) If using normal approximation, they must either write this or $\frac{7.5-4}{2}$ or $\frac{7.5-4}{\sqrt{3.84}}$ or $\frac{7.5-4}{\text{awrt } 1.96}$ or $\frac{7.5-20}{\sqrt{16}}$	M1
	= 1 – 0.9489 = 0.0511		A1 awrt 0.0511	A1

Question Number	Scheme	Notes	Marks
5(a)	X~Po(4) P(X = 0) = 0.0183 P($X \geq 8$) = 0.0511 P($X \leq 1$) = 0.0916 P($X \geq 9$) = 0.0214	M1: using Po(4), need to see a probability from Po(4), need not be one of the 4 given here. May be implied by a single correct CR	M1 A1 A1
	CR $X = 0$ $X \geq 9$	A1: $X = 0$ or $X \leq 0$ or $X < 1$ A1: $X \geq 9$ or $X > 8$ Any letter(s) may be used instead of X eg CR or Y or in words **SC** candidates who write P(X = 0) and $\mathrm{P}(X \geq 9)$ award M1A1 A0 **NB** Candidates who write $8 < x \leq 0$ oe get M1A0A0	
(b)	$\mathrm{H}_0 : \lambda = 4 \quad \mathrm{H}_1 : \lambda \neq 4$	B1: both hypotheses correct, labelled H_0 or NH or H_n and H_1 or AH or H_a may use λ or μ. These must be seen in part (b)	B1
	There is evidence that ***Liftsforall's*** claim is true **or** There is insufficient evidence to doubt ***Liftforall's*** claim	B1: ft their CR only, Do not ft hypotheses.Needs to include the word ***Liftsforall.*** If no Critical region stated in part (a) award B0 or $\mathrm{P}(X \leq 3) =$ awrt 0.434 and a correct conclusion.	B1ft
(c)	0.0183 + 0.0214 = 0.0397	B1: Awrt 0.0397	B1
(d)	P($B \leq 3$| B ~ Po(6)) = 0.1512	M1: using Po(6) and writing or using P($B \leq 3$) oe. A1: awrt 0.151	M1 A1
	X ~ B(4, 0.1512)	B1ft: dep on M1 being awarded. Using or writing B(4,"their 0.151") for use they need $(1-p)^4$ or $p(1-p)^3$ or $p^2(1-p)^2$	dB1ft
	Alternative method for first 3 marks		
	P($B \geq 4$| B ~ Po(6)) = 0.8488	M1: using Po(6) and writing or using P($B \geq 4$) oe A1: awrt 0.849	M1 A1
	Y ~ B(4, 0.849)	B1ft: dep on M1 being awarded. Using or writing B(4,"their 0.849") for use they need $(p)^4$ or $p^3(1-p)$ or $p^2(1-p)^2$	dB1ft
	If 0 < *p* < 0.5		
	P($X \leq 1$) = P(X = 0) + P(X = 1)	M1: using or writing P(X = 0) + P(X = 1) oe	M1
	$(1-0.1512)^4 + 4\times(1-0.1512)^3 \times 0.1512$	M1: $(1-p)^4 + 4\times(1-p)^3 \times p$ oe	dM1
	= 0.889	A1: awrt 0.889	A1
	If 0.5 < *p* < 1		
	P($Y \geq 3$) = P(Y = 3) + P(Y = 4)	M1: using or writing P(X = 3) + P(X = 4) oe	M1
	$4\times(0.8488)^3 \times 0.1512 + (0.8488)^4$	M1: $(p)^4 + 4\times(p)^3 \times(1-p)$ oe	dM1
	= 0.889	A1: awrt 0.889	A1

NB: a correct answer implies full marks, lose the final A mark if got awrt 0.889 and go on to do more work

Question Number	Scheme		Marks
	NB: All powers of 1 **must** be simplified for the Accuracy(A) marks		
		notes	
6(a)	$\left[\dfrac{kx^{n+1}}{n+1}\right]_0^1 = 1$	M1: attempting to integrate $x^n \to x^{n+1}$ and putting equal to 1, ignore limits A1: correct integration	M1A1
	$k = n + 1$	A1: $k = n + 1$ Do **not** accept $\dfrac{n+1}{1^{n+1}}$	A1

(b)	$\int_0^1 kx^{n+1}\mathrm{d}x = \left[\dfrac{kx^{n+2}}{n+2}\right]_0^1$	M1: Writing or using $\int_0^1 kx^{n+1}\mathrm{d}x$, ignore limits. Allow $\int_0^1 kx(x)^n\mathrm{d}x$ Allow substitution of their k A1: correct integration $\dfrac{kx^{n+2}}{n+2}$	M1A1
	$= \dfrac{n+1}{n+2}$	A1: correct answer only- must be in terms or n	A1cao

(c)	$\int_0^1 kx^{n+2}\mathrm{d}x = \left[\dfrac{kx^{n+3}}{n+3}\right]$	M1: Attempting to integrate $\int_0^1 kx^{n+2}\mathrm{d}x$, $x^{n+2} \to x^{n+3}$, ignore limits. Do not allow substitution of k if it has x in it. This must be on its own with no extra bits added on.	M1 A1cao
	$= \dfrac{n+1}{n+3}$	A1: correct answer only SC if they have $\dfrac{k}{n+2}$ as answer to part(b) award A1 for $\dfrac{k}{n+3}$	

(d)	Var (X) = $\dfrac{3}{5} - \left(\dfrac{3}{4}\right)^2 = \dfrac{3}{80}$	M1: using "their(c)"– ["their(b)"]^2 with $n = 2$ or correct Var(X) Using $\int_0^1 kx^4\mathrm{d}x - \left[\int_0^1 kx^3\mathrm{d}x\right]^2$ for Var(X)	M1
	Var (3X) = 9 Var (X)	M1: for writing or using 9 Var (X) or 3^2Var(X)	M1 A1cso
	$= \dfrac{27}{80}$ oe or 0.3375 or 0.338	A1: cso	

Question Number	Scheme	Notes	Marks
7	**NB: If there is a fully correct table award full marks.**		
	P(10) = 0.2, P(20) = 0.4 and P(50) = 0.4	B1: using P(10) = 0.2 (p) P(20) = 0.4(q) and P(50) = 0.4(r) may be seen in calculations or implied by a correct probability.	B1
	Median 10, 20, 50	B1: three correct medians and no extras.	B1
	P(Median 10) = $0.2^3 + 3\times 0.2^2\times 0.4 + 3\times 0.2^2\times 0.4$ **or** $0.2^3 + 3\times 0.2^2\times 0.8$	M1: allow if $(p+q+r)$=1 and use $p^3 + 3\times p^2\times q + 3\times p^2\times r$ **or** $p^3 + 3\times p^2\times (q+r)$ look for $\frac{1}{125}+\frac{6}{125}+\frac{6}{125}$	See below for how to award
	P(Median 50) = $0.4^3 + 3\times 0.4^2\times 0.2 + 3\times 0.4^2\times 0.4$ **or** $0.4^3 + 3\times 0.4^2\times 0.6$	M1: allow if $(p+q+r)$=1 and use $r^3 + 3\times r^2\times p + 3\times r^2\times q$ **or** $r^3 + 3\times r^2\times (p+q)$ Look for $\frac{8}{125}+\frac{12}{125}+\frac{24}{125}$	
	P(Median 20) = $3\times 0.2\times 0.4^2 + 6\times 0.2\times 0.4\times 0.4 + 0.4^3 + 3\times 0.4^2\times 0.4$	M1: allow if $(p+q+r)$=1 and use $3\times p\times q^2 + 6\times p\times q\times r + q^3 + 3\times q^2\times r$ $\frac{12}{125}+\frac{24}{125}+\frac{8}{125}+\frac{24}{125}$	
	<u>**How to award the M marks**</u> **– Allow the use of 1, 2 and 5 for the medians for the method marks** **M1** any correct calculation (implied by correct answer) for P(m = 10) or P(m = 20) or P(m = 50) **M1** any 2 correct calculations (implied by 2 correct answers) P(m = 10) or P(m = 20) or P(m = 50) **M1** any 3 correct calculations (implied by 3 correct answers) for P(m = 10) and P(m = 20) and P(m = 50) or 3 probabilities that add up to 1 providing it is 1 – their 2 other calculated probabilities. Do **not** allow $\frac{1}{5}$ $\frac{2}{5}$ $\frac{2}{5}$ **NB** if they do not have a correct answer their working must be clear including the addition signs.		
	<table><tr><td>median</td><td>10</td><td>20</td><td>50</td></tr><tr><td></td><td>0.104</td><td>0.544</td><td>0.352</td></tr><tr><td></td><td>Or $\frac{13}{125}$</td><td>Or $\frac{68}{125}$</td><td>Or $\frac{44}{125}$</td></tr></table>	A1: awrt any 1 correct A2: awrt all 3 correct These do not need to be in a table as long as the correct probablity is with the correct median(10, 20 & 50) **NB: Do Not allow the use of 1,2 and 5 for the medians for the A marks**	A2